MMAT5010 Linear Analysis (2022-23): Homework 3. Deadline: 16 Oct 2022

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let $\mathbb{K}^n$ be a n-dimension column vector space. Let A be a $n \times n$ matrix. Show that the map $x \in \mathbb{K}^n \mapsto Ax \in \mathbb{K}^n$ is continuous with respect to any norm $\|\cdot\|$ defined on $\mathbb{K}^n$.

2. Suppose that $\|\cdot\|_1$ and $\|\cdot\|_2$ are two equivalents norms defined on a vector space X. Let A be a subset of X. Show that if A is compact with respect to the norm $\|\cdot\|_1$, then A is also compact with respect to the norm $\|\cdot\|_2$.

3. Show that if (x_n) is a convergent sequence in ℓ_1, then it is also a convergent sequence with respect to the $\|\cdot\|_\infty$. Give an example of a sequence to show that the converse of this statement is not true.

$* * *$ **End** $* * *$